A STUDY OF THE TEMPERATURE BOUNDARY LAYER AND THE TRANSFER OF HEAT FOR AN ISOTHERMAL FLAT PLATE UNDER CONDITIONS OF TRANSVERSE STREAMLINING BY A LOW-TURBULENCE STREAM OF AIR

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We present the results of an experimental study dealing with the thermal characteristics of the boundary layer on an isothermal flat plate under conditions of transverse streamlining.

A number of experimental projects [1–3] have dealt with the investigation of heat transfer in the case of the transverse streamlining of a plate. However, in none of these projects was sufficient attention devoted to the turbulence of the free stream, whereas this factor is one of the most decisive [4,5]. In addition, these projects dealt with a variety of problems, as a result of which the complex and uniform conditions of wall isothermicity were not maintained, nor were the conditions of the wall's plane geometry, or the uniformity of the free stream. Although these differences are insignificant, the results were nevertheless markedly divergent (see Fig. 3). The theoretical solutions have limited application, since they have been derived only for the forward point and its vicinity [6,7].

The experimental installation (Fig. 1) consisted of a low-turbulence wind tunnel 1 [8], an automatic device 2 to measure the temperature fields in the boundary layer [9], an experimental heat exchanger 3, and thermostating systems 4–10. All of the quantitative conclusions regarding heat transfer were based on the results obtained in measuring the local temperature gradients at the wall, these having been determined from an analysis of the boundary layer. The selection of the method was based on its ability to yield results that are free of the influence of heat leakage, and whether or not it will make it possible to study the transfer of heat over the entire surface of the disk, including its edge. The method was analyzed from the standpoint of its effect on such factors as radiative heat exchange, the linear interpolation of temperature distribution at the wall, the asymmetry of the conditions of heat transfer between the thermocouple and the ambient medium, and the thermal inertia of the thermocouple. Moreover, the results of the automatic measurements were compared with data derived especially for this purpose by a calorimetric method.

The radiative heat exchange between the wall and the thermocouple in the heat-flow direction under consideration should fundamentally distort the slop of the temperature-distribution curve in the direction of the declining gradient. The quantitative evaluation of this effect was achieved experimentally. The thermocouple was mounted over the boundary layer for this purpose and it was then shifted along the disk radius. It thus found itself under conditions of maximum and virtually constant radiative heat exchange, but with differing convection conditions. Since reference [8] has provided us with the law governing the change in the velocity of the potential flow along the radius, we were able to determine the quantitative relationship between the additional heating of the thermocouple (which resulted in its thermal balance) and the velocity of the medium by which the thermocouple was streamlined. It was established that for the maximum temperature difference which occurred in these studies between the medium and the disk at 75° C, the greatest additional heating was found at the center of the disk, i.e., under conditons of minimum local velocity, and it amounted to 2° C (2.5% of the total temperature head). From a local velocity of 1 m/sec, there was absolutely no additional heating. Consequently, the region at the center of the disk—where the error attributable to the radiant flux was of the indicated order—was extremely limited and involved only hundredths of a fraction of the radius when the free-stream velocities were substantial.

It proved to be impossible to determine the true temperature distribution at the very surface of the disk, since the dimensions of the thermocouple itself prevented this. The temperature distribution at this point was interpolated linearly on the basis of the previous points and on the basis of the point corresponding to the temperature and the position of the wall. The wall temperature was determined from the thermocouple imbedded within it, and the position was determined from the instant at which the tip came into contact. Basically, such an interpolation should lead to a reduction in the gradient being determined for the wall. But since the actual temperature distribution in the immediate vicinity of the wall was close to the linear, this reduction can be neglected.

The thermal inertia of a thermocouple moving constantly across the boundary layer changes in accordance with the concurrence variations in the convection conditions. When moving toward the wall (as is specified in the automatic device) and with approach to the wall, the thermal inertia of the thermocouple increases and the temperature of the thermocouple must increasingly lag behind the rising temperature of the ambient medium. Basically, this lag must result in a distortion of the temperature profiles being measured. However, experimental verification of the thermal inertia of the thermocouple by shutting down the automatic device for a period of time (for the assumed warming up of the thermocouple) revealed no changes in the readings after the device was started up again. This

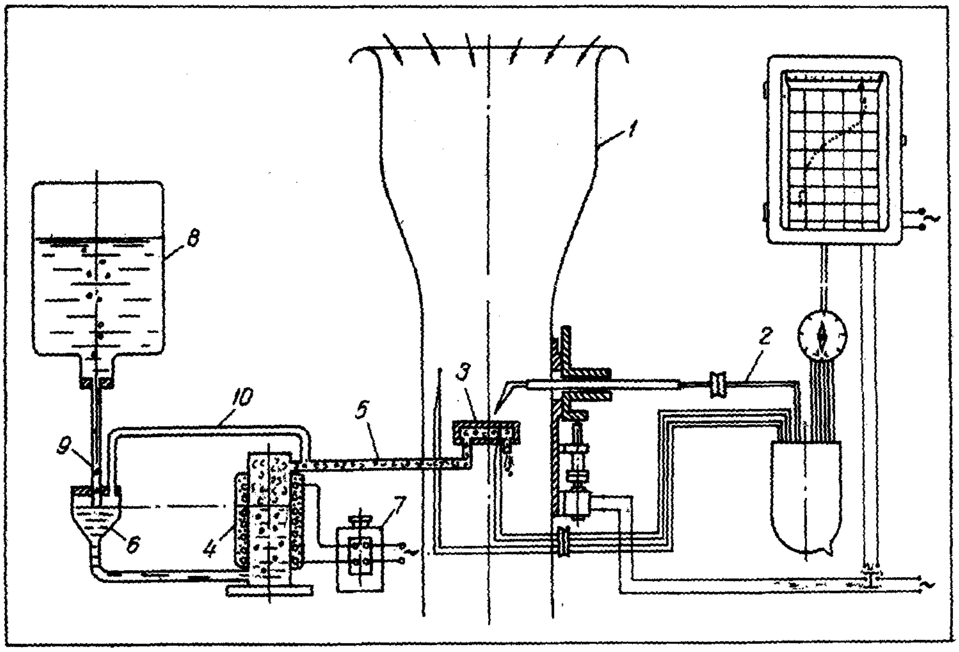

Fig. 1. Scheme of experimental installation.

indicated the practical absence of thermal inertia on the part of the thermocouple.

It was assumed in the measurements that the recorded readings of the thermocouple pertain to the medium at the points corresponding to the instantaneous positions of the geometric center of the thermocouple junction. As a matter of fact, because of the large temperature gradient in the boundary layer, and owing to the fact that the diameter of the thermocouple junction is commensurate with the thickness of the boundary layer, individual sections of the junction surface found themselves under various temperature conditions. It is obvious that under these conditions the mean thermocouple-junction temperature must be closer to that of the medium of the side on which the heat-transfer coefficient is higher. From the data of [10] we can conclude for the lateral streamlining of a cylinder in the case of low Reynolds numbers that with regard to the direction of the local-velocity vector this side will

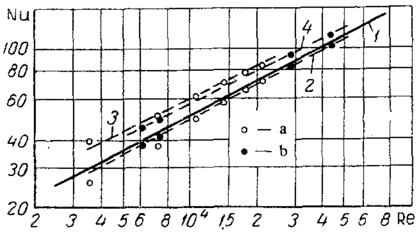

Fig. 2. Comparison of results from various methods of studying mean heat transfer in the central part of the disk within the range of linear velocity increase; 1) calculation according to [6] based on data of [8]; 2) results for temperature gradients; 3 and 4) calorimetric measurements for disk D = 100 (a) and 200 mm (b).

be the forward side in the case of a thermocouple, while under conditions of nonsymmetric streamlining in the boundary layer, it will be that portion of the frontal surface that is streamlined with a greater velocity, i.e., that portion which is turned toward the external flow. Consequently, for the heat-flux direction under consideration the readings of the thermocouple were somewhat lower than the temperature corresponding to the geometric position of its center. This was not a constant difference and varied in proportion to approach of the wall as the thermocouple was shifted in the lateral direction; this was a result of the change in the temperature gradient and in velocity. The temperature profile determined from these measurements should consequently be slightly distorted. The appearance of such a distortion in the temperature profiles was easily detected as the profiles were deflected from the point characterizing the wall temperature and position on the diagram. The measurements showed that the deflection is significant only in the central region of the disk and that this is obviously due to the fact that the surface at this point of the forward side of the thermocouple is turned entirely to face the external flow. The profiles experiencing this type of distortion were not considered in

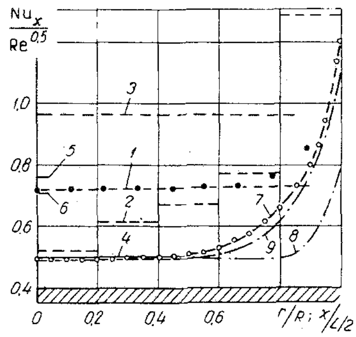

Fig. 3. Investigation results for local heat transfer on a plate in transverse flow: 1) tests [1] (plate); 2) tests [2] (disk); 3) tests [3] at Re = $= 10^4$ (disk); 4) calculations according to [6] based on data from [8]; 5) solution of [7] (disk); 6) the same corrected with the account for data from [8]; 7) authors' experiments; 8) calculation according to [11] based on data of [8]; 9) the same with authors' assumptions.

the processing of the results. As the free-stream velocity was increased, the boundary of this region was reduced. This factor did not become evident in the remaining sections of the boundary layer. The critical point remained that one point at which this type of deformation of the profile did not disappear, regardless of the regime. However, an investigation of the qualitative aspects of the temperature profiles by means of an IZK-454 interferometer showed that the thickness

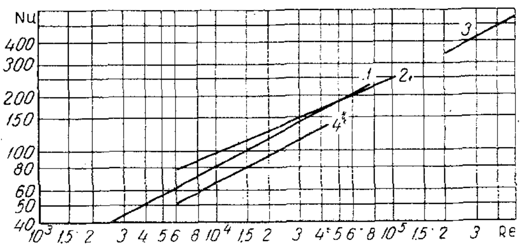

Fig. 4. Experimental data on mean heat transfer for a plate in transverse flow: 1) data of [2]; 2) data of [3]; 3) data of [1]; 4) data of the present study with respect to relation (2).

of the boundary layer and of the temperature profile over the entire central region remained virtually identical, up to r/R = 0.5. This last confirmed in experiments especially conducted for this purpose by the equality in this region of the local intensity of naphthalene sublimation.

Thus the measurement results which served as the basis for our conclusions could contain only those errors which were due to the linear interpolation of the temperature at the wall and of the radiative heating of the thermocouple. In principle, consequently, the measured temperature gradient could thus only be understated. In quantitative terms, this understatement is insignificant.

On the other hand, exaggerated results are characteristic of the calorimetric method under certain conditions, as a result of various types of heat losses. By using the two methods we were able to compare their data and to find additional arguments refining the results derived with the automatic device. On the basis of the second method, we fabricated two calorimetric heat exchanges with electrical heating and a temperature shield. The designs of these exchangers revealed our effort to reduce heat losses as much as possible, and they made it possible to determine the averaged heat transfer in the vicinity of the critical point by direct measurement of the power supplied. This last fact made it possible to compare the experimental data with the exact solution existing for this region [6]. The results of the investigation into the hydrodynamic boundary layer [8] showed that the boundary of the region of the critical point under the conditions being considered here extends virtually up to $r/R = 0.5$. This determined the selection of the relationship between the heated surfaces of the calorimeters and their unheated annular envelopes. Structurally, the heat exchanges were identical and differed only in their diameters (100 and 200 mm). Without presenting descriptions of their designs, we should point out that the complete shielding of the inoperative surfaces was impossible, as a result of which some loss of heat from the heated portion toward the unheated textolite envelope did occur. The unshielded surface of the larger disk was relatively smaller, and this should be taken into consideration in a critical evaluation of the results achieved with this method. Let us take notice of the fact that the radiant losses which were reduced to negligibly small values by thorough polishing of the surfaces also basically served to exaggerate the results. The installation of the larger disk resulted in the blockage of the tunnel beyond conventionally accepted limits. This led us to recall the conclusions of [8] as to the insignificant effect of blockage on the hydrodynamic streamlining of the central portion of the disk in the special case of transverse streamlining. Let us note that the change in the nature of the streamlining of the disk because of the blockage resulted in an increase in the heat transfer, which served also to increase the overstatement of the data. The nonisothermicity of the shielded disks was insignificant and amounted to a fraction of a degree. In addition to other factors, this was explained by the relative constancy of the heat-transfer coefficient in the central region of the disk.

The results from the measurement of the mean heat-transfer coefficient in the vicinity of the critical point, derived with the aid of the calorimetric heat exchangers, and the results obtained by averaging the local coefficients determined for this region from the temperature fields are given in Fig. 2 in the form of the function $Nu = f(Re)$. The mean temperature boundary layer was taken as the decisive temperature here and in all the subsequent experimental relationships. Comparison of the results from the various methods showed that the data derived from the temperature gradients at the various disks coincide and fall on a single curve (Fig. 2, curve 2). However, the results obtained by means of calorimetry (Fig. 2, curves 3 and 4) differ from each other and are higher than the first, while the slopes of these curves are less steep. It is noteworthy that this increase in the absolute expression of mean heat-transfer coefficient for each disk was of a different magnitude, which remained approximately constant in each case for all velocity and temperature regimes. The increase for the larger disk proved to be smaller here. Analysis of these circumstances, with consideration of the relationships for the unshielded portions of the heat exchangers, indicates some heat losses, predominantly of the conduction type, which were relatively smaller in the case of the larger disk. It is obvious that with an increase in the flow velocity and in convection heat transfer the relative magnitude of these losses diminished, so that the result should approximate the true magnitudes. Consequently, the fact that losses were present, and the approach of each of the curves obtained with this method to the curve plotted for the averaged local coefficients serves as additional confirmation of the fact that the chosen method of determining the local heat transfer on the basis of the temperature fields is precise. This is also confirmed by the closeness of the results to the data from the exact solution of the boundary-layer equations [6] (Fig. 2, curve 1). On the basis of all of the foregoing, if we neglect the effect of the factors under consideration, the accuracy of the results (with the exception of those for the forward point) is governed by the precision classification of the instruments used in determining the flow velocity, the calibration of the thermocouples, and the measurement of their readings, as well as by the error introduced in the processing of the results. For reliable values of the velocity regimes, the measurement error did not exceed $\pm 4\%$.

For a complete investigation of heat transfer over the entire extent of the radius we employed an unshielded disk 100 mm in diameter, heated with saturated steam. The transition to another heating method was necessitated by the fact that under conditions of pronounced nonuniform heat removal from the streamlined surface the electrical heaters were unable to provide the required isothermicity.

The system of thermostating the heat exchanger with saturated steam (see Fig. 1) made it possible to specify and automatically to maintain an operating regime for steam generator 4 such as to achieve a certain superheating of the steam. This was necessary only to prevent the formation of a condensate in connection tube 5 on the path to heat exchanger 3. The specification of the regime was achieved by changing the position of the communicating vessel 6 with respect to the steam generator and by regulating the laboratory autotransformer 7 of the supplied electrical power. This specified level was maintained automatically by feeding the system from the overturned reservoir 8 at the instant at which drainage tube 9 was opened as the level of water dropped. The condensate which appeared during the period of system heating was removed as a result of the increasing pressure within the system. Leveling tube 10 served to prevent disruption of system operation.

With the change in the flow velocity from 2 to 14 m/sec, the complete range of study covered $6.5 \cdot 10^3 \leq \mathrm{Re} \leq 4.5 \cdot 10^4$. The results of the complete investigation of the local heat-transfer coefficients referred to the entire disk surface—averaged for several velocity regimes—are given in Fig. 3, curve 7. We see from the result that in the case under consideration the distribution of the reference local heat-transfer coefficients is universal and a function exclusively of the relative radius r/R. We can isolate two characteristic regions here: the central region (virtually up to r/R = 0.5), where the heat transfer may be regarded as remaining constant and characteristic of the vicinity of the critical point, and a peripheral region in which the intensity of the heat transfer is markedly increased. Figure 3 presents the data of other authors [1–3, 6, 7] for purposes of comparison, as well as the results obtained from calculation by an approximate method [11] and those obtained from the experimental data of [8]. We were also able to obtain the solution of Sibulkin [6] on the basis of the experimental data in [8]. The figure also shows the Motulevich [7] solution which we corrected on the basis of the data in [8]. This correction enabled us to free the solution of the inaccuracy resulting from the theoretical-potential velocity distribution from [12], which he had used. In turn, this made it possible to undertake a more correct comparison of the Motulevich solution with the results of the exact solution according to Sibulkin [6], as well as with the results of our experiment and, thus, to evaluate the remaining divergence resulting from the retention of other assumptions in the Motulevich solution. It is noteworthy that the results of this investigation, in good agreement with the exact solution of the boundary-layer equations of [6], are lower from the quantitative standpoint than the results of all known experimental investigations. It is possible that this may be explained exclusively by the accuracy of the selected method, or possibly also by the low turbulence of the flow in our experiments. The following circumstance is worthy of attention. If in the method used in [11] for the transition to the calculation of heat transfer we adopt the tabulated frictional form parameter at the wall and the complex consisting of local values of the friction factor and the Reynolds and Nusselt numbers, constant over the extent of the entire boundary layer, and precisely such as established at the critical point, the results of such a calculation will yield excellent agreement with experiment (see Fig. 3, curve 9).

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The generalized values of the local heat-transfer coefficients are satisfactorily approximated by the relationship

$$\mathrm{Nu}_x/\mathrm{Re}^{0.5} = 0.5 \div 0.7 \cdot (r/R)^7. \quad (1)$$

The Nusselt number, calculated from the mean heat-transfer coefficient for the entire disk surface, is approximated by

$$\mathrm{Nu} = 0.65\,\mathrm{Re}^{0.5}, \quad (2)$$

derived by integration and subsequent averaging of relationship (1). Relationship (2) is shown graphically in Fig. 4, curve 4. Since the derived results pertain to the laminar boundary layer, their applicability is governed by the limits of the existence of the latter. The calculation carried out in accordance with the method in [13], in conjunction with the use of the hydrodynamic characteristics derived in [8], showed that under conditions of transverse streamlining for a flat disk by a low-turbulence medium the laminar boundary layer is stable up to $R = 10^9$.

NOTATION

D and R are the diameter and radius of the disk; r is the instantaneous radius; L is the plate width in [1]; x is the distance from the point of flow bifurcation in [1]; U_∞ is the velocity of the incoming flow; U is the velocity in the external boundary layer; ν is the coefficient of kinematic viscosity; y is the coordinate normal to disk; t_f is the temperature of incoming flow; t_w is the wall temperature; t is the temperature in the boundary layer; $\bar{t}$ is the temperature in the boundary layer in dimensionless form; α is the mean heat transfer coefficient; α_x is the local mean heat transfer coefficient; λ is the thermal conductivity; Nu is the mean Nusselt number; Nu_x is the local Nusselt number; Re is the Reynolds number.

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